Journal of Statistical Physics, Vol. 82, Nos. 1/2, 1996

Book Review: *The Statistical Mechanics of Lattice Gases*

The Statistical Mechanics of Lattice Gases. Vol. 1. B. Simon, Princeton University Press, Princeton, New Jersey, 1993.

Graduate students often asks questions like: What exactly is a phase transition? When is it first or second order? Does phase transition mean: Nonanalyticity of the thermodynamic functions? Coexistence of several phases? (And what is the precise definition of a phase?) Or some slow decay of correlation functions? Is it equivalent to spontaneous symmetry breakdown? One of the achievements of mathematical statistical mechanics is to give precise (but not obvious) answers to those questions. Not only are there exact definitions for all these concepts, but they can be rather easily illustrated by several examples and counterexamples.

In fact, the list of achievements is rather impressive: if I limit myself to classical spin systems, and to rigorous results, we have a general formalism covering the notion of equilibrium states and of phase transitions, a very detailed understanding of the high-temperature phase, an almost complete understanding of the low-temperature one (at least for discrete spins) due to Pirogov and Sinai, a good theory of the spontaneous breakdown of continuous symmetries and of Kosterlitz–Thouless transitions, and several results on the critical point, either in low dimension (through exact solutions) or in high dimension (via correlation inequalities or renormalization group methods). What is more, the general formalism, as well as several special techniques, have sometime surprising connections with such diverse areas as quantum field theory, the ergodic theory of dynamical systems, percolation theory, and Markov processes.

However, it is not easy to find a complete exposition of this theory which is pedagogical enough to be accessible to graduate students. The goal of this two-volume work by Barry Simon (of which only the first is available now) is to fill this gap. This book, like other works of the same author, is well written, with the right mixture of rigorous mathematics, motivations, and examples.

0022-4715/96/0100-0453$09.50/0

The content reflects a logical order: first, define the thermodynamic limit for the pressure and related quantities, like the entropy, and connect them via the variational principle. Introduce the different notions of equilibrium states (solutions of the DLR equations, solutions of the variational principle, tangents to the pressure) and show when and in what sense they are equivalent. Extend this to quantum spin systems. Then study the high-temperature or low-density phase where the equilibrium state is unique and the decay of the correlation functions is well understood. Then, move on to the "sexiest" topics, as Simon calls them: phase transitions, infrared bounds, the Fröhlich–Spencer theory, and correlation inequalities. Unfortunately, the reader will have to wait for Volume 2 in order to learn about these topics. In fact, the main problem with this book is that it took too long to come out. It all started with a course in 1979–1980! Fortunately, this delay does not make the book outdated (although some relatively recent developments, like the Dobrushin–Shlosman theory, are not included), since most of the material is quite classical. Let us just hope that it will not take so long for the second volume to appear.

One of the nice characteristics of this book is that the exposition of the general thermodynamic formalism is sprinkled with examples and applications: transfer matrices (including the solution of the two-dimensional Ising model), duality, mean field theory, absence of phase transition in one dimension, and Mermin–Wagner-type theorems (including the complex translation method of McBryan and Spencer), among others. The high-temperature phase is analyzed both via Dobrushin's methods (with detailed bounds on critical temperatures) and via polymer expansions (however, I did not find that this last part was optimally well written).

In summary, this excellent text will be used both as a reference book by experts (each topic is covered in great detail) and, skipping some sections, as the basis of an advanced graduate course. It can also safely be given to graduate students who want an answer to the questions mentioned in the beginning of this review.

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